Linear combinations of Vectors

$\underline{a} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

Then $2\underline{a} = 2\begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$

$\underline{b} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ $\quad 3\underline{b} = 3\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ 9 \end{pmatrix}$

$2\underline{a} + 3\underline{b}$ is called a Linear combination of vectors.

$$2\underline{a} + 3\underline{b} = \begin{pmatrix} 6 \\ 2 \end{pmatrix} + \begin{pmatrix} 6 \\ 9 \end{pmatrix}$$

$$= \underline{\underline{\begin{pmatrix} 12 \\ 11 \end{pmatrix}}}$$

Example

If $\underline{a} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$ and $\underline{c} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, find a vector $\underline{b}$ so that

$$2\underline{a} + \underline{b} = 3\underline{c}$$

$$2\begin{pmatrix} 3 \\ -1 \end{pmatrix} + \begin{pmatrix} x \\ y \end{pmatrix} = 3\begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} 6 \\ -2 \end{pmatrix} + \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ 9 \end{pmatrix}$$

Both 'rows' must be true.

$6 + x = 6$ so $x = 0$

$-2 + y = 9$ so $y = 11$

So $\underline{b} = \begin{pmatrix} 0 \\ 11 \end{pmatrix}$

Any vector that has an x value of zero will be parallel to the y axis.

Any vector that has a y value of zero will be parallel to the x axis.

Example

If $\underline{p} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and $\underline{q} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, find values of l and m so that

$l\underline{p} + m\underline{q}$ is parallel to the x-axis

$$l\begin{pmatrix} 3 \\ 2 \end{pmatrix} + m\begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} x \\ 0 \end{pmatrix}$$

$$2l + 3m = 0$$

Let $\underline{\underline{l = 10}}$

$$2 \times 10 + 3m = 0$$

$$20 + 3m = 0$$

$$3m = -20$$

$$\underline{\underline{m = \frac{-20}{3}}}$$

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Let $\underline{\underline{l = 3}}$

$$2 \times 3 + 3m = 0$$

$$6 + 3m = 0$$

$$3m = -6$$

$$\underline{\underline{m = -2}}$$